

**QUESTION PAPER CODE 65/1/1/D**  
**EXPECTED ANSWER/VALUE POINTS**  
**SECTION A**

1.  $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin \theta & 0 \\ 1 & 0 & \cos \theta \end{vmatrix} = \sin \theta \cos \theta$   $\frac{1}{2}$
- $= \frac{1}{2} \sin 2\theta \therefore \text{Max value} = \frac{1}{2}$   $\frac{1}{2}$
2.  $(A - I)^3 + (A + I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$   $\frac{1}{2}$
- $= 2A - A = A$   $\frac{1}{2}$
3.  $\left. \begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{array} \right\}$   $\frac{1}{2} + \frac{1}{2}$
4. Getting position vector as  $2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})$   $\frac{1}{2}$
- $= 3\vec{a} + 4\vec{b}$   $\frac{1}{2}$
5.  $\overline{AD} = \overline{AB} + \frac{1}{2}[\overline{AC} - \overline{AB}] = \frac{1}{2}(\overline{AC} + \overline{AB})$   $\frac{1}{2}$
- $|\overline{AD}| = \frac{1}{2}|3\hat{i} + 5\hat{k}| = \frac{1}{2}\sqrt{34}$   $\frac{1}{2}$
6.  $\vec{r} \cdot \frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7} = 5$  1

**SECTION B**

7. LHS =  $\tan^{-1} \left( \frac{\frac{1}{5} + \frac{1}{7}}{1 - \frac{1}{5} \cdot \frac{1}{7}} \right) + \tan^{-1} \left( \frac{\frac{1}{3} + \frac{1}{8}}{1 - \frac{1}{3} \cdot \frac{1}{8}} \right)$  1
- $= \tan^{-1} \left( \frac{6}{17} \right) + \tan^{-1} \left( \frac{11}{23} \right)$  1
- $= \tan^{-1} \left( \frac{\frac{6}{17} + \frac{11}{23}}{1 - \frac{6}{17} \cdot \frac{11}{23}} \right) = \tan^{-1} \left( \frac{325}{325} \right)$  1
- $= \tan^{-1} (1) = \frac{\pi}{4}$  1

OR

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right) \quad 2$$

$$\Rightarrow \sin x (\sin x - \cos x) = 0 \quad 1$$

$$\Rightarrow \sin x = \cos x \quad \frac{1}{2}$$

$$\text{the solution is } x = \frac{\pi}{4} \quad \frac{1}{2}$$

8. Let the income be  $3x$ ,  $4x$  and expenditures,  $5y$ ,  $7y$

$$\therefore \left. \begin{array}{l} 3x - 5y = 15000 \\ 4x - 7y = 15000 \end{array} \right\} \quad 1$$

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1 \begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\Rightarrow x = 30000, y = 15000 \quad 1\frac{1}{2}$$

$$\therefore \text{Incomes are ₹ 90000 and ₹ 120000 respectively} \quad \frac{1}{2}$$

“Expenditure must be less than income”

(or any other relevant answer) 1

9. Here  $x = a\left(\sin 2t + \frac{1}{2}\sin 4t\right)$ ,  $y = b(\cos 2t - \cos^2 2t)$

$$\frac{dx}{dt} = 2a[\cos 2t + \cos 4t], \frac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t] \quad 1 + 1$$

$$\frac{dy}{dx} = \frac{b}{a} \left[ \frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t} \right] \quad 1$$

$$\left. \frac{dy}{dx} \right]_{t=\frac{\pi}{4}} = \frac{b}{a} \quad \frac{1}{2}$$

$$\text{and } \left. \frac{dy}{dx} \right]_{t=\frac{\pi}{3}} = \sqrt{3} \frac{b}{a} \quad \frac{1}{2}$$

OR

$$y = x^x \Rightarrow \log y = x \cdot \log x \quad \frac{1}{2}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = (1 + \log x) \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{1}{y} \frac{d^2y}{dx^2} - \frac{1}{y^2} \left( \frac{dy}{dx} \right)^2 = \frac{1}{x} \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0 \quad \frac{1}{2}$$

$$10. \text{ LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{(1 - \cancel{\sin x})(1 + \sin x + \sin^2 x)}{3(1 - \cancel{\sin x})(1 + \sin x)} \quad 1$$

$$= \frac{1}{2} \quad \frac{1}{2}$$

$$\therefore p = \frac{1}{2} \quad \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{q(1 - \sin x)}{(\pi - 2x)^2} = \lim_{h \rightarrow 0} \frac{q(1 - \cos h)}{(2h)^2}, \text{ where } x - \frac{\pi}{2} = h \quad 1$$

$$= \lim_{h \rightarrow 0} \frac{2q \sin^2 \frac{h}{2}}{4.4. \frac{h^2}{4}} = \frac{q}{8} \quad \frac{1}{2}$$

$$\therefore \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4 \quad \frac{1}{2}$$

$$11. \frac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3 \sin t (1 - \cos^2 t) = -3 \sin^3 t \quad 1$$

$$\frac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t (1 - \sin^2 t) = 3\cos^3 t \quad 1$$

$$\text{Slope of normal} = -\frac{dx}{dy} = \frac{\sin^3 t}{\cos^3 t} \quad 1$$

Eqn. of normal is

$$y - (3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t} [x - (3\cos t - \cos^3 t)] \quad \frac{1}{2}$$

$$\Rightarrow y \cos^3 t - x \sin^3 t = 3\sin t \cos t (\cos^2 t - \sin^2 t)$$

$$= \frac{3}{4} \sin 4t \quad \frac{1}{2}$$

$$\text{or } 4(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$$

$$12. I = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - (1 - \sin^2 \theta) - 4 \sin \theta} d\theta \quad \frac{1}{2}$$

$$\sin \theta = t \Rightarrow \cos \theta d\theta = dt$$

$$\therefore I = \int \frac{3t - 2}{t^2 - 4t + 4} dt = \int \frac{3t - 2}{(t - 2)^2} dt \quad 1$$

$$= \int \frac{3(t-2)}{(t-2)^2} dt + 4 \int \frac{1}{(t-2)^2} dt \quad 1$$

$$= 3 \log |t-2| - \frac{4}{(t-2)} + C \quad 1$$

$$= 3 \log |\sin \theta - 2| - \frac{4}{(\sin \theta - 2)} + C \quad \frac{1}{2}$$

**OR**

$$\text{Let } I = \int_0^\pi \sin\left(\frac{\pi}{4} + x\right) e^{2x} dx$$

$$= \sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} \Big|_0^\pi - \int_0^\pi \cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx \quad 1$$

$$I = \left[ \sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} - \frac{1}{2} \left\{ \cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} \right\} \right]_0^\pi + \frac{1}{2} \int_0^\pi -\sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx \quad 1$$

$$\frac{5}{4} I = \left\{ \frac{1}{4} \left[ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right] e^{2x} \right\}_0^\pi \quad 1$$

$$I = \frac{1}{5} \left[ \left\{ 2 \left( -\frac{1}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \right\} e^{2\pi} - \left\{ 2 \left( \frac{1}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}} \right\} \right] = \frac{-1}{5\sqrt{2}} (e^{2\pi} + 1) \quad 1$$

$$13. \quad I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$$

$$\text{Put } x^{3/2} = t \Rightarrow \frac{3}{2} x^{1/2} dx = dt \text{ or } \sqrt{x} dx = \frac{2}{3} dt \quad 1 \frac{1}{2}$$

$$I = \frac{2}{3} \int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} \quad 1$$

$$= \frac{2}{3} \cdot \sin^{-1} \left( \frac{t}{a^{3/2}} \right) + C \quad 1$$

$$= \frac{2}{3} \sin^{-1} \left( \frac{x^{3/2}}{a^{3/2}} \right) + C \quad \frac{1}{2}$$

$$14. \quad I = \int_{-1}^2 |x^3 - x| dx$$

$$= \int_{-1}^0 (x^3 - x) dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) dx \quad 1 \frac{1}{2}$$

$$= \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 + \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_1^2 \quad 1 \frac{1}{2}$$

$$= -\left( \frac{1}{4} - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{4} \right) + (4 - 2) - \left( \frac{1}{4} - \frac{1}{2} \right)$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4} \quad 1$$

15. Given differential equation can be written as

$$\frac{(1 + \log x)}{x} dx + \frac{2y}{1 - y^2} dy = 0 \quad 1$$

$$\text{integrating to get, } \frac{1}{2} (1 + \log x)^2 - \log |1 - y^2| = C \quad 2$$

$$x = 1, y = 0 \Rightarrow C = \frac{1}{2} \quad \frac{1}{2}$$

$$\Rightarrow (1 + \log x)^2 - 2 \log |1 - y^2| = 1 \quad \frac{1}{2}$$

16. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1 + y^2} x = \frac{e^{\tan^{-1} y}}{1 + y^2} \quad 1$$

Integrating factor is  $e^{\tan^{-1} y}$  1

$$\therefore \text{ Solution is } x \cdot e^{\tan^{-1} y} = \int e^{2 \tan^{-1} y} \frac{1}{1 + y^2} dy \quad 1$$

$$\therefore x e^{\tan^{-1} y} = \frac{1}{2} e^{2 \tan^{-1} y} + C \quad 1$$

17. Given, that  $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$  are coplanar

$$\therefore [\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 0$$

$$\text{i.e. } (\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\} = 0 \quad 1$$

$$(\vec{a} + \vec{b}) \cdot \{(\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a})\} = 0 \quad 1$$

$$\Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a}) = 0 \quad 1 \frac{1}{2}$$

$$\Rightarrow 2[\vec{a}, \vec{b}, \vec{c}] = 0 \text{ or } [\vec{a}, \vec{b}, \vec{c}] = 0 \quad \frac{1}{2}$$

$$\Rightarrow \vec{a}, \vec{b}, \vec{c} \text{ are coplanar.}$$

18. Vector equation of the required line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu [(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})] \quad 1$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda [(2\hat{i} + 3\hat{j} + 6\hat{k})] \quad 2$$

$$\text{in cartesian form, } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \quad 1$$

19. Let events are:

$$\left. \begin{array}{l} E_1 : A \text{ is selected} \\ E_2 : B \text{ is selected} \\ E_3 : C \text{ is selected} \\ A : \text{Change is not introduced} \end{array} \right\}$$

$$P(E_1) = \frac{1}{7} \quad P(E_2) = \frac{2}{7} \quad P(E_3) = \frac{4}{7}$$

1

$$P(A/E_1) = 0.2, \quad P(A/E_2) = 0.5, \quad P(A/E_3) = 0.7$$

1

$$\begin{aligned} \therefore P(E_3/A) &= \frac{\frac{4}{7} \times \frac{7}{10}}{\frac{1}{7} \times \frac{2}{10} + \frac{2}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{7}{10}} \\ &= \frac{28}{40} = \frac{7}{10} \end{aligned}$$

1

1

OR

$$\left. \begin{array}{l} \text{Prob. of success for A} = \frac{1}{6} \\ \text{Prob. of failure for A} = \frac{5}{6} \\ \text{Prob. of success for B} = \frac{1}{12} \\ \text{Prob. of failure for B} = \frac{11}{12} \end{array} \right\}$$

1

B can win in 2nd or 4th or 6th or....throw

1

$$\begin{aligned} \therefore P(B) &= \left( \frac{5}{6} \frac{1}{12} \right) + \left( \frac{5}{6} \frac{11}{12} \frac{5}{6} \frac{1}{12} \right) + \left( \frac{5}{6} \frac{11}{12} \frac{5}{6} \frac{11}{12} \frac{5}{6} \frac{1}{12} \right) + \dots \\ &= \frac{5}{72} \left( 1 + \frac{55}{72} + \left( \frac{55}{72} \right)^2 + \dots \right) \\ &= \frac{5}{72} \times \frac{1}{1 - \frac{55}{72}} = \frac{5}{72} \times \frac{72}{17} = \frac{5}{17} \end{aligned}$$

1

1

## SECTION C

20. Let  $x_1, x_2 \in \mathbb{N}$  and  $f(x_1) = f(x_2)$ 

$$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2$$

$$\Rightarrow (x_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$$

$$\Rightarrow x_1 - x_2 = 0 \text{ or } x_1 = x_2 \text{ as } (9x_1 + 9x_2 + 6) \neq 0, x_1, x_2 \in \mathbb{N}$$

$\therefore f$  is a one-one function

2

$f: \mathbb{N} \rightarrow \mathbb{S}$  is ONTO as co-domain = Range

1

Hence  $f$  is invertible

$$y = 9x^2 + 6x - 5 = (3x + 1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$$

$$\therefore f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}, \quad y \in \mathbb{S}$$

2

$$f^{-1}(43) = \frac{\sqrt{49}-1}{3} = 2$$

1  
2

$$f^{-1}(163) = \frac{\sqrt{169}-1}{3} = 4$$

1  
2

21. Using  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$  we get

$$\Delta = \begin{vmatrix} y(z-x) + z^2 - x^2 & x(z-y) + z^2 - y^2 & xy - z^2 \\ z(x-y) + x^2 - y^2 & y(x-z) + x^2 - z^2 & yz - x^2 \\ x(y-z) + y^2 - z^2 & z(y-x) + y^2 - x^2 & zx - y^2 \end{vmatrix} \quad 2$$

Taking  $(x + y + z)$  common from  $C_1$  &  $C_2$

$$\Rightarrow \Delta = (x + y + z)^2 \begin{vmatrix} z-x & z-y & xy - z^2 \\ x-y & x-z & yz - x^2 \\ y-z & y-x & zx - y^2 \end{vmatrix} \quad 1$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \Delta = (x + y + z)^2 \begin{vmatrix} 0 & 0 & xy + yz + zx - x^2 - y^2 - z^2 \\ x-y & x-z & yz - x^2 \\ y-z & y-x & zx - y^2 \end{vmatrix} \quad 1$$

Expanding to get

$$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2 \quad 1$$

Hence  $\Delta$  is divisible by  $(x + y + z)$  and

$$\text{the quotient is } (x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2 \quad 1$$

**OR**

$$\text{Writing} \quad \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad 1$$

$$R_1 \leftrightarrow R_3 \quad \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$$

$$\begin{matrix} R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 - 4R_2 \end{matrix} \quad \begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$$

$$\begin{matrix} R_1 \rightarrow \frac{1}{3}R_1 \\ R_3 \rightarrow -R_3 \end{matrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$$

$$R_2 \rightarrow R_2 - 2R_1 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A \quad 2\frac{1}{2} \text{ marks for operation to get } A^{-1}$$

$$R_2 \rightarrow R_2 - R_3 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$$

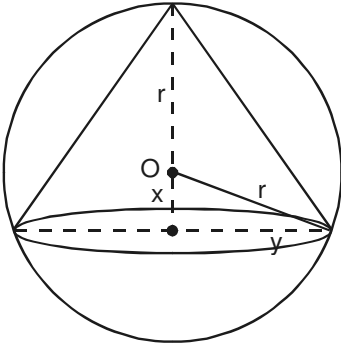
$$\therefore A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \quad 1$$

$$AX = B \Rightarrow X = A^{-1}B \quad 1$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\therefore x = 1, y = 2, z = 1 \quad 1$$

22.



Correct Figure

1

Let radius of cone be  $y$  and the altitude be  $r + x$ 

$$\therefore x^2 + y^2 = r^2 \quad \dots(i)$$

 $\frac{1}{2}$ 

$$\text{Volume } V = \frac{1}{3} \pi y^2 (r + x)$$

$$= \frac{1}{3} \pi (r^2 - x^2) (r + x)$$

1

$$\frac{dV}{dx} = \frac{\pi}{3} [(r^2 - x^2)1 + (r + x)(-2x)] = \frac{\pi}{3} (r + x)(r - 3x)$$

1

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3}$$

 $\frac{1}{2}$ 

$$\therefore \text{Altitude} = r + \frac{r}{3} = \frac{4r}{3}$$

 $\frac{1}{2}$ 

$$\text{and } \frac{d^2V}{dx^2} = \frac{\pi}{3} [(r + x)(-3) + (r - 3x)] = \frac{\pi}{3} [-2r - 6x] < 0$$

1

$$\therefore \text{Max. Volume} = \frac{\pi}{3} \left( r^2 - \frac{r^2}{9} \right) \left( r + \frac{r}{3} \right) = \frac{8}{27} \left( \frac{4}{3} \pi r^3 \right)$$

 $\frac{1}{2}$ 

$$= \frac{8}{27} (\text{Vol. of sphere})$$

**OR**

$$f(x) = \sin 3x - \cos 3x, \quad 0 < x < \pi$$

$$f'(x) = 3 \cos 3x + 3 \sin 3x$$

1

$$f'(x) = 0 \Rightarrow \tan 3x = -1$$

 $\frac{1}{2}$ 

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{4}, \quad n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

 $\frac{1}{2}$ 

$$\text{Intervals are: } \left( 0, \frac{\pi}{4} \right), \left( \frac{\pi}{4}, \frac{7\pi}{12} \right), \left( \frac{7\pi}{12}, \frac{11\pi}{12} \right), \left( \frac{11\pi}{12}, \pi \right)$$

1

$$f(x) \text{ is strictly increasing in } \left( 0, \frac{\pi}{4} \right) \cup \left( \frac{7\pi}{12}, \frac{11\pi}{12} \right)$$

1

$$\text{and strictly decreasing in } \left( \frac{\pi}{4}, \frac{7\pi}{12} \right) \cup \left( \frac{11\pi}{12}, \pi \right)$$

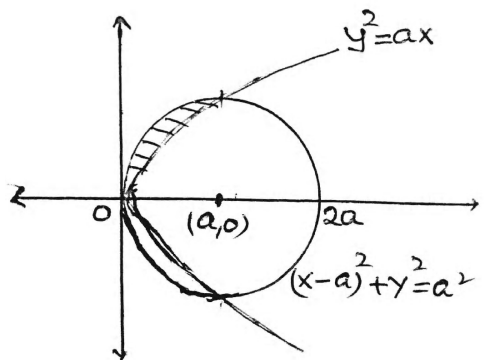
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23.

$$y^2 = ax, x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$$

$$\Rightarrow x = 0, x = a$$



Correct Figure

$$\text{Shaded area} = \left[ \int_0^a [\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}] dx \right]$$

$$A = \left[ \frac{x-a}{2} \sqrt{a^2 - (x-a)^2} + \frac{a^2}{2} \sin^{-1} \frac{x-a}{a} - \sqrt{a} \frac{2}{3} x^{3/2} \right]_0^a$$

$$= \left[ -\frac{2}{3} a^2 + \frac{a^2}{2} \frac{\pi}{2} \right] = \frac{\pi a^2}{4} - \frac{2a^2}{3} \text{ sq. units}$$

24. Equation of line AB:  $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$

Eqn. of plane LMN:  $\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$

$$2(x-2) + 1(y-2) + 1(z-1) = 0 \text{ or } 2x + y + z - 7 = 0$$

Any point on line AB is  $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$

If this point lies on plane, then  $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$

$\therefore$  P is  $(1, -2, 7)$

let P divides AB in K : 1

$$\Rightarrow 1 = \frac{2K+3}{K+1} \Rightarrow K = -2 \text{ i.e. P divides, AB externally in } 2:1$$

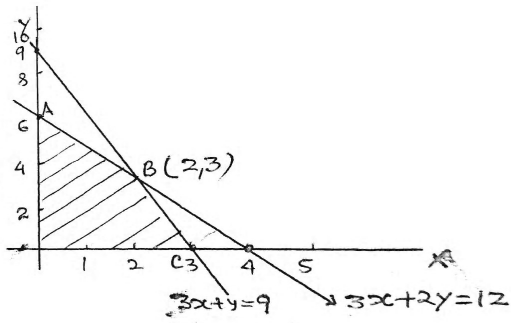
25. X = No. of red

X:	0	1	2	3	4
P(X):	${}^4C_0 \left(\frac{1}{3}\right)^4$ $= \frac{1}{81}$	${}^4C_1 \left(\frac{1}{3}\right)^3 \frac{2}{3}$ $= \frac{8}{81}$	${}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$ $= \frac{24}{81}$	${}^4C_3 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^3$ $= \frac{32}{81}$	${}^4C_4 \left(\frac{2}{3}\right)^4$ $= \frac{16}{81}$
XP(X):	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$
X <sup>2</sup> P(X):	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$

$$\text{Mean} = \Sigma XP(X) = \frac{216}{81} = \frac{8}{3}$$

$$\text{Variance} = \Sigma X^2P(X) - [\Sigma XP(X)]^2 = \frac{648}{81} - \left(\frac{8}{3}\right)^2 = \frac{64}{9} - \frac{64}{9} = \frac{8}{9}$$

26.

Let production of A, B (per day) be  $x, y$  respectively

Maximise  $P = 7x + 4y$

$$\text{Subject to } \left. \begin{array}{l} 3x + 2y \leq 12 \\ 3x + y \leq 9 \\ x \geq 0, y \geq 0 \end{array} \right\}$$

Correct Graph

$P(A) = 24$

$P(B) = 26$

$P(C) = 21$

 $\therefore$  2 units of product A and 3 units of product B for maximum profit

QUESTION PAPER CODE 65/1/2/D  
**EXPECTED ANSWER/VALUE POINTS**  
**SECTION A**

1. 
$$\left. \begin{aligned} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{aligned} \right\}$$
  $\frac{1}{2} + \frac{1}{2}$
2. Getting position vector as  $2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})$   $\frac{1}{2}$   

$$= 3\vec{a} + 4\vec{b}$$
  $\frac{1}{2}$
3.  $\vec{AD} = \vec{AB} + \frac{1}{2}[\vec{AC} - \vec{AB}] = \frac{1}{2}(\vec{AC} + \vec{AB})$   $\frac{1}{2}$   
 $|\vec{AD}| = \frac{1}{2}|3\hat{i} + 5\hat{k}| = \frac{1}{2}\sqrt{34}$   $\frac{1}{2}$
4.  $\vec{r} \cdot \frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7} = 5$  1
5.  $\Delta = \begin{vmatrix} 0 & 0 \\ \sin \theta & 0 \\ 0 & \cos \theta \end{vmatrix} = \sin \theta \cos \theta$   $\frac{1}{2}$   
 $= \frac{1}{2} \sin 2\theta \therefore \text{Max value} = \frac{1}{2}$   $\frac{1}{2}$
6.  $(A - I)^3 + (A + I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$   $\frac{1}{2}$   
 $= 2A - A = A$   $\frac{1}{2}$

**SECTION B**

7.  $\frac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3 \sin t (1 - \cos^2 t) = -3 \sin^3 t$  1
- $\frac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t (1 - \sin^2 t) = 3\cos^3 t$  1
- Slope of normal =  $-\frac{dx}{dy} = \frac{\sin^3 t}{\cos^3 t}$  1
- Eqn. of normal is
- $y - (3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t} [x - (3\cos t - \cos^3 t)]$   $\frac{1}{2}$
- $\Rightarrow y \cos^3 t - x \sin^3 t = 3\sin t \cos t (\cos^2 t - \sin^2 t)$
- $= \frac{3}{4} \sin 4t$   $\frac{1}{2}$
- or  $4(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$

$$8. \quad I = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - (1 - \sin^2 \theta) - 4 \sin \theta} d\theta \quad \frac{1}{2}$$

$$\sin \theta = t \Rightarrow \cos \theta d\theta = dt$$

$$\therefore \quad I = \int \frac{3t - 2}{t^2 - 4t + 4} dt = \int \frac{3t - 2}{(t - 2)^2} dt \quad 1$$

$$= \int \frac{3(t - 2)}{(t - 2)^2} dt + 4 \int \frac{1}{(t - 2)^2} dt \quad 1$$

$$= 3 \log |t - 2| - \frac{4}{(t - 2)} + C \quad 1$$

$$= 3 \log |\sin \theta - 2| - \frac{4}{(\sin \theta - 2)} + C \quad \frac{1}{2}$$

**OR**

$$\text{Let } I = \int_0^\pi \sin\left(\frac{\pi}{4} + x\right) e^{2x} dx$$

$$= \sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} \Big|_0^\pi - \int_0^\pi \cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx \quad 1$$

$$I = \left[ \sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} - \frac{1}{2} \left\{ \cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} \right\} \right]_0^\pi + \frac{1}{2} \int_0^\pi -\sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx \quad 1$$

$$\frac{5}{4} I = \left\{ \frac{1}{4} \left[ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right] e^{2x} \right\}_0^\pi \quad 1$$

$$I = \frac{1}{5} \left[ \left\{ 2 \left( -\frac{1}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \right\} e^{2\pi} - \left\{ 2 \left( \frac{1}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}} \right\} \right] = \frac{-1}{5\sqrt{2}} (e^{2\pi} + 1) \quad 1$$

$$9. \quad I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$$

$$\text{Put } x^{3/2} = t \Rightarrow \frac{3}{2} \cdot x^{1/2} dx = dt \text{ or } \sqrt{x} dx = \frac{2}{3} dt \quad \frac{1}{2}$$

$$I = \frac{2}{3} \int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} \quad 1$$

$$= \frac{2}{3} \cdot \sin^{-1} \left( \frac{t}{a^{3/2}} \right) + C \quad 1$$

$$= \frac{2}{3} \sin^{-1} \left( \frac{x^{3/2}}{a^{3/2}} \right) + C \quad \frac{1}{2}$$

$$10. \quad I = \int_{-1}^2 |x^3 - x| dx$$

$$= \int_{-1}^0 (x^3 - x) dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) dx \quad \frac{1}{2}$$

$$\begin{aligned}
&= \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 + \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_1^2 \\
&= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right) \\
&= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}
\end{aligned}$$

1  
 $\frac{1}{2}$ 

1

11. Given differential equation can be written as

$$\frac{(1 + \log x)}{x} dx + \frac{2y}{1 - y^2} dy = 0 \quad 1$$

$$\text{integrating to get, } \frac{1}{2} (1 + \log x)^2 - \log |1 - y^2| = C \quad 2$$

$$x = 1, y = 0 \Rightarrow C = \frac{1}{2} \quad \frac{1}{2}$$

$$\Rightarrow (1 + \log x)^2 - 2 \log |1 - y^2| = 1 \quad \frac{1}{2}$$

12. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1 + y^2} x = \frac{e^{\tan^{-1} y}}{1 + y^2} \quad 1$$

$$\text{Integrating factor is } e^{\tan^{-1} y} \quad 1$$

$$\therefore \text{ Solution is } x \cdot e^{\tan^{-1} y} = \int e^{2 \tan^{-1} y} \frac{1}{1 + y^2} dy \quad 1$$

$$\therefore x e^{\tan^{-1} y} = \frac{1}{2} e^{2 \tan^{-1} y} + C \quad 1$$

13. Given, that  $\vec{a} + \vec{b}$ ,  $\vec{b} + \vec{c}$ ,  $\vec{c} + \vec{a}$  are coplanar

$$\therefore [\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 0$$

$$\text{i.e. } (\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\} = 0 \quad 1$$

$$(\vec{a} + \vec{b}) \cdot \{(\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a})\} = 0 \quad 1$$

$$\Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a}) = 0 \quad 1 \frac{1}{2}$$

$$\Rightarrow 2[\vec{a}, \vec{b}, \vec{c}] = 0 \text{ or } [\vec{a}, \vec{b}, \vec{c}] = 0 \quad \frac{1}{2}$$

$$\Rightarrow \vec{a}, \vec{b}, \vec{c} \text{ are coplanar.}$$

14. Vector equation of the required line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu [(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})] \quad 1$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda [(2\hat{i} + 3\hat{j} + 6\hat{k})] \quad 2$$

$$\text{in cartesian form, } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \quad 1$$

15. Let events are:

$$\left. \begin{array}{l} E_1 : A \text{ is selected} \\ E_2 : B \text{ is selected} \\ E_3 : C \text{ is selected} \\ A : \text{Change is not introduced} \end{array} \right\}$$

$$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7} \quad 1$$

$$P(A/E_1) = 0.2, P(A/E_2) = 0.5, P(A/E_3) = 0.7 \quad 1$$

$$\begin{aligned} \therefore P(E_3/A) &= \frac{\frac{4}{7} \times \frac{7}{10}}{\frac{1}{7} \times \frac{2}{10} + \frac{2}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{7}{10}} \quad 1 \\ &= \frac{28}{40} = \frac{7}{10} \quad 1 \end{aligned}$$

**OR**

$$\left. \begin{array}{l} \text{Prob. of success for A} = \frac{1}{6} \\ \text{Prob. of failure for A} = \frac{5}{6} \\ \text{Prob. of success for B} = \frac{1}{12} \\ \text{Prob. of failure for B} = \frac{11}{12} \end{array} \right\} \quad 1$$

B can win in 2nd or 4th or 6th or....throw 1

$$\begin{aligned} \therefore P(B) &= \left( \frac{5}{6} \cdot \frac{1}{12} \right) + \left( \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12} \right) + \left( \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12} \right) + \dots \quad 1 \\ &= \frac{5}{72} \left( 1 + \frac{55}{72} + \left( \frac{55}{72} \right)^2 + \dots \right) \\ &= \frac{5}{72} \times \frac{1}{1 - \frac{55}{72}} = \frac{5}{72} \times \frac{72}{17} = \frac{5}{17} \quad 1 \end{aligned}$$

$$\begin{aligned} 16. \text{ LHS} &= \tan^{-1} \left( \frac{\frac{1}{5} + \frac{1}{7}}{1 - \frac{1}{5} \cdot \frac{1}{7}} \right) + \tan^{-1} \left( \frac{\frac{1}{3} + \frac{1}{8}}{1 - \frac{1}{3} \cdot \frac{1}{8}} \right) \quad 1 \\ &= \tan^{-1} \left( \frac{6}{17} \right) + \tan^{-1} \left( \frac{11}{23} \right) \quad 1 \\ &= \tan^{-1} \left( \frac{\frac{6}{17} + \frac{11}{23}}{1 - \frac{6}{17} \cdot \frac{11}{23}} \right) = \tan^{-1} \left( \frac{325}{325} \right) \quad 1 \\ &= \tan^{-1} (1) = \frac{\pi}{4} \quad 1 \end{aligned}$$

OR

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right) \quad 2$$

$$\Rightarrow \sin x (\sin x - \cos x) = 0 \quad 1$$

$$\Rightarrow \sin x = \cos x \quad \frac{1}{2}$$

$$\text{the solution is } x = \frac{\pi}{4} \quad \frac{1}{2}$$

17. Let the income be  $3x$ ,  $4x$  and expenditures,  $5y$ ,  $7y$

$$\therefore \left. \begin{array}{l} 3x - 5y = 15000 \\ 4x - 7y = 15000 \end{array} \right\} \quad 1$$

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1 \begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\Rightarrow x = 30000, y = 15000 \quad 1\frac{1}{2}$$

$$\therefore \text{Incomes are ₹ 90000 and ₹ 120000 respectively} \quad \frac{1}{2}$$

“Expenditure must be less than income”

(or any other relevant answer) 1

18. Here  $x = a\left(\sin 2t + \frac{1}{2}\sin 4t\right)$ ,  $y = b(\cos 2t - \cos^2 2t)$

$$\frac{dx}{dt} = 2a[\cos 2t + \cos 4t], \frac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t] \quad 1 + 1$$

$$\frac{dy}{dx} = \frac{b}{a} \left[ \frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t} \right] \quad 1$$

$$\left. \frac{dy}{dx} \right]_{t=\frac{\pi}{4}} = \frac{b}{a} \quad \frac{1}{2}$$

$$\text{and } \left. \frac{dy}{dx} \right]_{t=\frac{\pi}{3}} = \sqrt{3} \frac{b}{a} \quad \frac{1}{2}$$

OR

$$y = x^x \Rightarrow \log y = x \cdot \log x \quad \frac{1}{2}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = (1 + \log x) \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{1}{y} \frac{d^2y}{dx^2} - \frac{1}{y^2} \left( \frac{dy}{dx} \right)^2 = \frac{1}{x} \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0 \quad \frac{1}{2}$$

19. LHL =  $\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{(1 - \sin x)(1 + \sin x + \sin^2 x)}{3(1 - \sin x)(1 + \sin x)} \quad 1$

$$= \frac{1}{2} \quad \frac{1}{2}$$

$$\therefore p = \frac{1}{2} \quad \frac{1}{2}$$

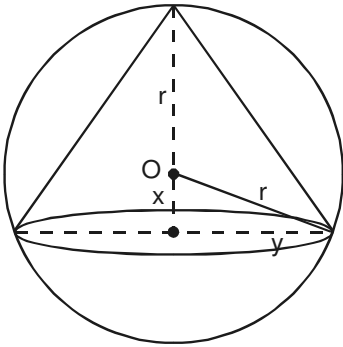
RHL =  $\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{q(1 - \sin x)}{(\pi - 2x)^2} = \lim_{h \rightarrow 0} \frac{q(1 - \cos h)}{(2h)^2}$ , where  $x - \frac{\pi}{2} = h \quad 1$

$$= \lim_{h \rightarrow 0} \frac{2q \sin^2 \frac{h}{2}}{4 \cdot \frac{h^2}{4}} = \frac{q}{8} \quad \frac{1}{2}$$

$$\therefore \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4 \quad \frac{1}{2}$$

## SECTION C

20.



Correct Figure 1

Let radius of cone be  $y$  and the altitude be  $r + x$ 

$$\therefore x^2 + y^2 = r^2 \quad \dots(i) \quad \frac{1}{2}$$

$$\begin{aligned} \text{Volume } V &= \frac{1}{3} \pi y^2 (r + x) \\ &= \frac{1}{3} \pi (r^2 - x^2) (r + x) \end{aligned} \quad 1$$

$$\frac{dV}{dx} = \frac{\pi}{3} [(r^2 - x^2)1 + (r + x)(-2x)] = \frac{\pi}{3} (r + x)(r - 3x) \quad 1$$

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3} \quad \frac{1}{2}$$

$$\therefore \text{Altitude} = r + \frac{r}{3} = \frac{4r}{3} \quad \frac{1}{2}$$

$$\text{and } \frac{d^2V}{dx^2} = \frac{\pi}{3} [(r + x)(-3) + (r - 3x)] = \frac{\pi}{3} [-2r - 6x] < 0 \quad 1$$

$$\begin{aligned} \therefore \text{Max. Volume} &= \frac{\pi}{3} \left( r^2 - \frac{r^2}{9} \right) \left( r + \frac{r}{3} \right) = \frac{8}{27} \left( \frac{4}{3} \pi r^3 \right) \quad \frac{1}{2} \\ &= \frac{8}{27} (\text{Vol. of sphere}) \end{aligned}$$



OR

$$f(x) = \sin 3x - \cos 3x, 0 < x < \pi$$

$$f'(x) = 3 \cos 3x + 3 \sin 3x$$

$$f'(x) = 0 \Rightarrow \tan 3x = -1$$

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{4}, n \in \mathbb{Z}$$

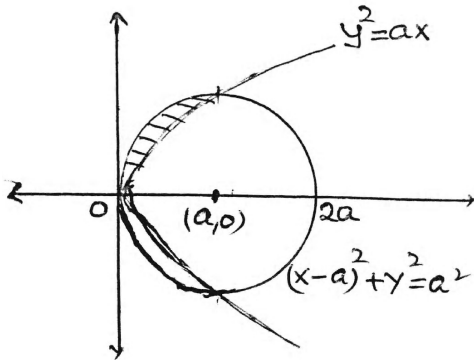
$$\Rightarrow x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

$$\text{Intervals are: } \left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{7\pi}{12}\right), \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right), \left(\frac{11\pi}{12}, \pi\right)$$

$$f(x) \text{ is strictly increasing in } \left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$$

$$\text{and strictly decreasing in } \left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$$

21.



$$y^2 = ax, x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$$

$$\Rightarrow x = 0, x = a$$

Correct Figure

$$\text{Shaded area} = \int_0^a [\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}] dx$$

$$A = \left[ \frac{x-a}{2} \sqrt{a^2 - (x-a)^2} + \frac{a^2}{2} \sin^{-1} \frac{x-a}{a} - \sqrt{a} \frac{2}{3} x^{3/2} \right]_0^a$$

$$= \left[ -\frac{2}{3} a^2 + \frac{a^2}{2} \frac{\pi}{2} \right] = \frac{\pi a^2}{4} - \frac{2a^2}{3} \text{ sq. units}$$

$$22. \text{ Equation of line AB: } \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$$

$$\text{Eqn. of plane LMN: } \begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$$

$$2(x-2) + 1(y-2) + 1(z-1) = 0 \text{ or } 2x + y + z - 7 = 0$$

$$\text{Any point on line AB is } (-\lambda + 3, \lambda - 4, 6\lambda - 5)$$

$$\text{If this point lies on plane, then } 2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$$

$$\therefore P \text{ is } (1, -2, 7)$$

let P divides AB in K : 1

$$\Rightarrow 1 = \frac{2K+3}{K+1} \Rightarrow K = -2 \text{ i.e. P divides, AB externally in } 2:1$$

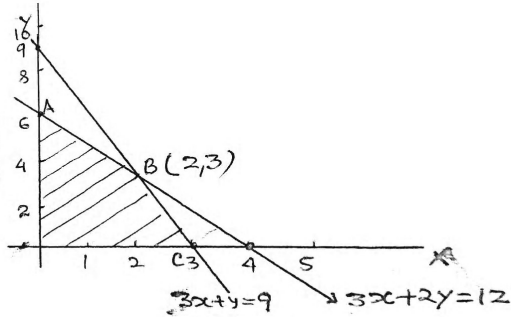
23. X = No. of red

X:	0	1	2	3	4	1
P(X):	${}^4C_0 \left(\frac{1}{3}\right)^4$ $= \frac{1}{81}$	${}^4C_1 \left(\frac{1}{3}\right)^3 \frac{2}{3}$ $= \frac{8}{81}$	${}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$ $= \frac{24}{81}$	${}^4C_3 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^3$ $= \frac{32}{81}$	${}^4C_4 \left(\frac{2}{3}\right)^4$ $= \frac{16}{81}$	$2\frac{1}{2}$
XP(X):	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$	
X <sup>2</sup> P(X):	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$	

Mean =  $\Sigma XP(X) = \frac{216}{81} = \frac{8}{3}$  1

Variance =  $\Sigma X^2P(X) - [\Sigma XP(X)]^2 = \frac{648}{81} - \left(\frac{64}{9}\right)^2 = \frac{8}{9}$   $1\frac{1}{2}$

24.



Let production of A, B (per day) be x, y respectively

Maximise  $P = 7x + 4y$  1

Subject to  $\left. \begin{aligned} 3x + 2y &\leq 12 \\ 3x + y &\leq 9 \\ x \geq 0, y &\geq 0 \end{aligned} \right\}$  2

Correct Graph 2

$P(A) = 24$

$P(B) = 26$

$P(C) = 21$

$\therefore$  2 units of product A and 3 units of product B for maximum profit 1

25. Let  $x_1, x_2 \in \mathbb{N}$  and  $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$

$\Rightarrow x_1 - x_2 = 0$  or  $x_1 = x_2$  as  $(9x_1 + 9x_2 + 6) \neq 0, x_1, x_2 \in \mathbb{N}$

$\therefore$  f is a one-one function 2

f:  $\mathbb{N} \rightarrow \mathbb{S}$  is ONTO as co-domain = Range 1

Hence f is invertible

$y = 9x^2 + 6x - 5 = (3x + 1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$

$\therefore f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}, y \in \mathbb{S}$  2

$f^{-1}(43) = \frac{\sqrt{49}-1}{3} = 2$   $\frac{1}{2}$

$f^{-1}(163) = \frac{\sqrt{169}-1}{3} = 4$   $\frac{1}{2}$

26. Using  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$  we get

$$\Delta = \begin{vmatrix} y(z-x) + z^2 - x^2 & x(z-y) + z^2 - y^2 & xy - z^2 \\ z(x-y) + x^2 - y^2 & y(x-z) + x^2 - z^2 & yz - x^2 \\ x(y-z) + y^2 - z^2 & z(y-x) + y^2 - x^2 & zx - y^2 \end{vmatrix} \quad 2$$

Taking  $(x + y + z)$  common from  $C_1$  &  $C_2$

$$\Rightarrow \Delta = (x + y + z)^2 \begin{vmatrix} z-x & z-y & xy - z^2 \\ x-y & x-z & yz - x^2 \\ y-z & y-x & zx - y^2 \end{vmatrix} \quad 1$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \Delta = (x + y + z)^2 \begin{vmatrix} 0 & 0 & xy + yz + zx - x^2 - y^2 - z^2 \\ x-y & x-z & yz - x^2 \\ y-z & y-x & zx - y^2 \end{vmatrix} \quad 1$$

Expanding to get

$$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2 \quad 1$$

Hence  $\Delta$  is divisible by  $(x + y + z)$  and

$$\text{the quotient is } (x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2 \quad 1$$

**OR**

$$\text{Writing} \quad \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad 1$$

$$R_1 \leftrightarrow R_3 \quad \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$$

$$\begin{matrix} R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 - 4R_2 \end{matrix} \quad \begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$$

$$\begin{matrix} R_1 \rightarrow \frac{1}{3}R_1 \\ R_3 \rightarrow -R_3 \end{matrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$$

$$R_2 \rightarrow R_2 - 2R_1 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A \quad 2\frac{1}{2} \text{ marks for operation to get } A^{-1}$$

$$R_2 \rightarrow R_2 - R_3 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$$

$$\therefore A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \quad 1$$

$$AX = B \Rightarrow X = A^{-1}B \quad 1$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\therefore x = 1, y = 2, z = 1 \quad 1$$

**QUESTION PAPER CODE 65/1/3/D**  
**EXPECTED ANSWER/VALUE POINTS**  
**SECTION A**

1.  $\overline{AD} = \overline{AB} + \frac{1}{2}[\overline{AC} - \overline{AB}] = \frac{1}{2}(\overline{AC} + \overline{AB})$   $\frac{1}{2}$
- $|\overline{AD}| = \frac{1}{2}|3\hat{i} + 5\hat{k}| = \frac{1}{2}\sqrt{34}$   $\frac{1}{2}$
2.  $\vec{r} \cdot \frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7} = 5$  1
3.  $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin \theta & 0 \\ 1 & 0 & \cos \theta \end{vmatrix} = \sin \theta \cos \theta$   $\frac{1}{2}$
- $= \frac{1}{2} \sin 2\theta \therefore \text{Max value} = \frac{1}{2}$   $\frac{1}{2}$
4.  $(A - I)^3 + (A + I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$   $\frac{1}{2}$
- $= 2A - A = A$   $\frac{1}{2}$
5.  $\left. \begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{array} \right\}$   $\frac{1}{2} + \frac{1}{2}$
6. Getting position vector as  $2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})$   $\frac{1}{2}$
- $= 3\vec{a} + 4\vec{b}$   $\frac{1}{2}$

**SECTION B**

7. Given differential equation can be written as
- $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$  1
- Integrating factor is  $e^{\tan^{-1}y}$  1
- $\therefore$  Solution is  $x \cdot e^{\tan^{-1}y} = \int e^{2\tan^{-1}y} \frac{1}{1+y^2} dy$  1
- $\therefore x e^{\tan^{-1}y} = \frac{1}{2} e^{2\tan^{-1}y} + C$  1
8. Given, that  $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$  are coplanar
- $\therefore [\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 0$

$$\text{i.e. } (\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\} = 0 \quad 1$$

$$(\vec{a} + \vec{b}) \cdot \{(\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a})\} = 0 \quad 1$$

$$\Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a}) = 0 \quad 1\frac{1}{2}$$

$$\Rightarrow 2[\vec{a}, \vec{b}, \vec{c}] = 0 \text{ or } [\vec{a}, \vec{b}, \vec{c}] = 0 \quad \frac{1}{2}$$

$$\Rightarrow \vec{a}, \vec{b}, \vec{c} \text{ are coplanar.}$$

9. Vector equation of the required line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu [(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})] \quad 1$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda [(2\hat{i} + 3\hat{j} + 6\hat{k})] \quad 2$$

$$\text{in cartesian form, } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \quad 1$$

10. Let events are:

$$\left. \begin{array}{l} E_1 : A \text{ is selected} \\ E_2 : B \text{ is selected} \\ E_3 : C \text{ is selected} \\ A : \text{Change is not introduced} \end{array} \right\}$$

$$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7} \quad 1$$

$$P(A/E_1) = 0.2, P(A/E_2) = 0.5, P(A/E_3) = 0.7 \quad 1$$

$$\therefore P(E_3/A) = \frac{\frac{4}{7} \times \frac{7}{10}}{\frac{1}{7} \times \frac{2}{10} + \frac{2}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{7}{10}} \quad 1$$

$$= \frac{28}{40} = \frac{7}{10} \quad 1$$

**OR**

$$\left. \begin{array}{l} \text{Prob. of success for A} = \frac{1}{6} \\ \text{Prob. of failure for A} = \frac{5}{6} \\ \text{Prob. of success for B} = \frac{1}{12} \\ \text{Prob. of failure for B} = \frac{11}{12} \end{array} \right\} \quad 1$$

B can win in 2nd or 4th or 6th or....throw 1

$$\therefore P(B) = \left(\frac{5}{6} \cdot \frac{1}{12}\right) + \left(\frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12}\right) + \left(\frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12}\right) + \dots \quad 1$$

$$= \frac{5}{72} \left( 1 + \frac{55}{72} + \left( \frac{55}{72} \right)^2 + \dots \right)$$

$$= \frac{5}{72} \times \frac{1}{1 - \frac{55}{72}} = \frac{5}{72} \times \frac{72}{17} = \frac{5}{17}$$

1

11. LHS =  $\tan^{-1} \left( \frac{\frac{1}{5} + \frac{1}{7}}{1 - \frac{1}{5} \cdot \frac{1}{7}} \right) + \tan^{-1} \left( \frac{\frac{1}{3} + \frac{1}{8}}{1 - \frac{1}{3} \cdot \frac{1}{8}} \right)$

1

$$= \tan^{-1} \left( \frac{6}{17} \right) + \tan^{-1} \left( \frac{11}{23} \right)$$

1

$$= \tan^{-1} \left( \frac{\frac{6}{17} + \frac{11}{23}}{1 - \frac{6}{17} \cdot \frac{11}{23}} \right) = \tan^{-1} \left( \frac{325}{325} \right)$$

1

$$= \tan^{-1} (1) = \frac{\pi}{4}$$

1

**OR**

$$2 \tan^{-1} (\cos x) = \tan^{-1} (2 \operatorname{cosec} x)$$

$$\Rightarrow \tan^{-1} \left( \frac{2 \cos x}{1 - \cos^2 x} \right) = \tan^{-1} \left( \frac{2}{\sin x} \right)$$

2

$$\Rightarrow \sin x (\sin x - \cos x) = 0$$

1

$$\Rightarrow \sin x = \cos x$$

 $\frac{1}{2}$ 

$$\text{the solution is } x = \frac{\pi}{4}$$

 $\frac{1}{2}$ 

12. Let the income be  $3x$ ,  $4x$  and expenditures,  $5y$ ,  $7y$

$$\therefore \left. \begin{array}{l} 3x - 5y = 15000 \\ 4x - 7y = 15000 \end{array} \right\}$$

1

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1 \begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\Rightarrow x = 30000, y = 15000$$

 $1\frac{1}{2}$ 

$\therefore$  Incomes are ₹ 90000 and ₹ 120000 respectively

 $\frac{1}{2}$ 

“Expenditure must be less than income”

(or any other relevant answer)

1

13. Here  $x = a \left( \sin 2t + \frac{1}{2} \sin 4t \right)$ ,  $y = b (\cos 2t - \cos^2 2t)$

$$\frac{dx}{dt} = 2a[\cos 2t + \cos 4t], \frac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t] \quad 1 + 1$$

$$\frac{dy}{dx} = \frac{b \left[ \frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t} \right]}{a} \quad 1$$

$$\left. \frac{dy}{dx} \right|_{t=\frac{\pi}{4}} = \frac{b}{a} \quad \frac{1}{2}$$

and  $\left. \frac{dy}{dx} \right|_{t=\frac{\pi}{3}} = \sqrt{3} \frac{b}{a} \quad \frac{1}{2}$

**OR**

$$y = x^x \Rightarrow \log y = x \cdot \log x \quad \frac{1}{2}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = (1 + \log x) \quad 1 \frac{1}{2}$$

$$\Rightarrow \frac{1}{y} \frac{d^2y}{dx^2} - \frac{1}{y^2} \left( \frac{dy}{dx} \right)^2 = \frac{1}{x} \quad 1 \frac{1}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0 \quad \frac{1}{2}$$

14. LHL =  $\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{(1 - \cancel{\sin x})(1 + \sin x + \sin^2 x)}{3(1 - \cancel{\sin x})(1 + \sin x)} \quad 1$

$$= \frac{1}{2} \quad \frac{1}{2}$$

$$\therefore p = \frac{1}{2} \quad \frac{1}{2}$$

RHL =  $\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{q(1 - \sin x)}{(\pi - 2x)^2} = \lim_{h \rightarrow 0} \frac{q(1 - \cos h)}{(2h)^2}$ , where  $x - \frac{\pi}{2} = h \quad 1$

$$= \lim_{h \rightarrow 0} \frac{2q \sin^2 \frac{h}{2}}{4 \cdot \frac{h^2}{4}} = \frac{q}{8} \quad \frac{1}{2}$$

$$\therefore \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4 \quad \frac{1}{2}$$

15.  $\frac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3 \sin t (1 - \cos^2 t) = -3 \sin^3 t \quad 1$

$$\frac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t (1 - \sin^2 t) = 3\cos^3 t \quad 1$$

$$\text{Slope of normal} = -\frac{dx}{dy} = \frac{\sin^3 t}{\cos^3 t} \quad 1$$

Eqn. of normal is

$$y - (3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t} [x - (3\cos t - \cos^3 t)] \quad \frac{1}{2}$$

$$\begin{aligned} \Rightarrow y \cos^3 t - x \sin^3 t &= 3\sin t \cos t (\cos^2 t - \sin^2 t) \\ &= \frac{3}{4} \sin 4t \end{aligned} \quad \frac{1}{2}$$

$$\text{or } 4(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$$

$$16. \quad I = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - (1 - \sin^2 \theta) - 4 \sin \theta} d\theta \quad \frac{1}{2}$$

$$\sin \theta = t \Rightarrow \cos \theta d\theta = dt$$

$$\therefore I = \int \frac{3t - 2}{t^2 - 4t + 4} dt = \int \frac{3t - 2}{(t - 2)^2} dt \quad 1$$

$$= \int \frac{3(t - 2)}{(t - 2)^2} dt + 4 \int \frac{1}{(t - 2)^2} dt \quad 1$$

$$= 3 \log |t - 2| - \frac{4}{(t - 2)} + C \quad 1$$

$$= 3 \log |\sin \theta - 2| - \frac{4}{(\sin \theta - 2)} + C \quad \frac{1}{2}$$

**OR**

$$\text{Let } I = \int_0^\pi \sin \left( \frac{\pi}{4} + x \right) e^{2x} dx$$

$$= \sin \left( \frac{\pi}{4} + x \right) \frac{e^{2x}}{2} \Big|_0^\pi - \int_0^\pi \cos \left( \frac{\pi}{4} + x \right) \frac{e^{2x}}{2} dx \quad 1$$

$$I = \left[ \sin \left( \frac{\pi}{4} + x \right) \frac{e^{2x}}{2} - \frac{1}{2} \left\{ \cos \left( \frac{\pi}{4} + x \right) \frac{e^{2x}}{2} \right\} \right]_0^\pi + \frac{1}{2} \int_0^\pi -\sin \left( \frac{\pi}{4} + x \right) \frac{e^{2x}}{2} dx \quad 1$$

$$\frac{5}{4} I = \left\{ \frac{1}{4} \left[ 2 \sin \left( \frac{\pi}{4} + x \right) - \cos \left( \frac{\pi}{4} + x \right) \right] e^{2x} \right\}_0^\pi \quad 1$$

$$I = \frac{1}{5} \left[ \left\{ 2 \left( -\frac{1}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \right\} e^{2\pi} - \left\{ 2 \left( \frac{1}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}} \right\} \right] = \frac{-1}{5\sqrt{2}} (e^{2\pi} + 1) \quad 1$$

$$17. \quad I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$$

$$\text{Put } x^{3/2} = t \Rightarrow \frac{3}{2} \cdot x^{1/2} dx = dt \text{ or } \sqrt{x} dx = \frac{2}{3} dt \quad 1 \frac{1}{2}$$



$$\begin{aligned}
 I &= \frac{2}{3} \int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} && 1 \\
 &= \frac{2}{3} \cdot \sin^{-1} \left( \frac{t}{a^{3/2}} \right) + C && 1 \\
 &= \frac{2}{3} \sin^{-1} \left( \frac{x^{3/2}}{a^{3/2}} \right) + C && \frac{1}{2}
 \end{aligned}$$

**18.**  $I = \int_{-1}^2 |x^3 - x| dx$

$$\begin{aligned}
 &= \int_{-1}^0 (x^3 - x) dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) dx && 1\frac{1}{2} \\
 &= \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 + \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_1^2 && 1\frac{1}{2} \\
 &= -\left( \frac{1}{4} - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{4} \right) + (4 - 2) - \left( \frac{1}{4} - \frac{1}{2} \right) \\
 &= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4} && 1
 \end{aligned}$$

**19.** Given differential equation can be written as

$$\frac{(1 + \log x)}{x} dx + \frac{2y}{1 - y^2} dy = 0 \quad 1$$

integrating to get,  $\frac{1}{2} (1 + \log x)^2 - \log |1 - y^2| = C$  2

$$x = 1, y = 0 \Rightarrow C = \frac{1}{2} \quad \frac{1}{2}$$

$$\Rightarrow (1 + \log x)^2 - 2 \log |1 - y^2| = 1 \quad \frac{1}{2}$$

### SECTION C

**20.** Equation of line AB:  $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$  1

Eqn. of plane LMN:  $\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$  1\frac{1}{2}

$$2(x - 2) + 1(y - 2) + 1(z - 1) = 0 \text{ or } 2x + y + z - 7 = 0 \quad \frac{1}{2}$$

Any point on line AB is  $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$  \frac{1}{2}

If this point lies on plane, then  $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$  1

$\therefore$  P is  $(1, -2, 7)$  \frac{1}{2}

let P divides AB in K : 1

$$\Rightarrow 1 = \frac{2K+3}{K+1} \Rightarrow K = -2 \text{ i.e. P divides, AB externally in 2:1}$$

1

21. X = No. of red

X:	0	1	2	3	4
P(X):	${}^4C_0 \left(\frac{1}{3}\right)^4$ $= \frac{1}{81}$	${}^4C_1 \left(\frac{1}{3}\right)^3 \frac{2}{3}$ $= \frac{8}{81}$	${}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$ $= \frac{24}{81}$	${}^4C_3 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^3$ $= \frac{32}{81}$	${}^4C_4 \left(\frac{2}{3}\right)^4$ $= \frac{16}{81}$
XP(X):	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$
X <sup>2</sup> P(X):	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$

1

 $2\frac{1}{2}$ 

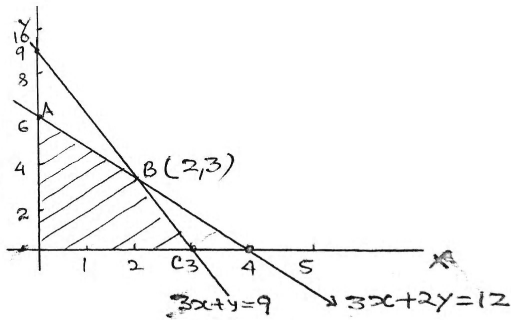
$$\text{Mean} = \Sigma XP(X) = \frac{216}{81} = \frac{8}{3}$$

1

$$\text{Variance} = \Sigma X^2P(X) - [\Sigma XP(X)]^2 = \frac{648}{81} - \frac{64}{9} = \frac{8}{9}$$

 $1\frac{1}{2}$ 

22.



Let production of A, B (per day) be x, y respectively

$$\text{Maximise } P = 7x + 4y$$

1

$$\text{Subject to } \left. \begin{array}{l} 3x + 2y \leq 12 \\ 3x + y \leq 9 \\ x \geq 0, y \geq 0 \end{array} \right\}$$

2

Correct Graph

2

$$P(A) = 24$$

$$P(B) = 26$$

$$P(C) = 21$$

$\therefore$  2 units of product A and 3 units of product B for maximum profit

1

23. Let  $x_1, x_2 \in \mathbb{N}$  and  $f(x_1) = f(x_2)$

$$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$$

$$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$$

$$\Rightarrow x_1 - x_2 = 0 \text{ or } x_1 = x_2 \text{ as } (9x_1 + 9x_2 + 6) \neq 0, x_1, x_2 \in \mathbb{N}$$

$\therefore$  f is a one-one function

2

f:  $\mathbb{N} \rightarrow \mathbb{S}$  is ONTO as co-domain = Range

1

Hence f is invertible

$$y = 9x^2 + 6x - 5 = (3x + 1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$$

$$\therefore f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}, y \in \mathbb{S}$$

2

$$f^{-1}(43) = \frac{\sqrt{49} - 1}{3} = 2 \quad \frac{1}{2}$$

$$f^{-1}(163) = \frac{\sqrt{169} - 1}{3} = 4 \quad \frac{1}{2}$$

24. Using  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$  we get

$$\Delta = \begin{vmatrix} y(z-x) + z^2 - x^2 & x(z-y) + z^2 - y^2 & xy - z^2 \\ z(x-y) + x^2 - y^2 & y(x-z) + x^2 - z^2 & yz - x^2 \\ x(y-z) + y^2 - z^2 & z(y-x) + y^2 - x^2 & zx - y^2 \end{vmatrix} \quad 2$$

Taking  $(x + y + z)$  common from  $C_1$  &  $C_2$

$$\Rightarrow \Delta = (x + y + z)^2 \begin{vmatrix} z-x & z-y & xy - z^2 \\ x-y & x-z & yz - x^2 \\ y-z & y-x & zx - y^2 \end{vmatrix} \quad 1$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \Delta = (x + y + z)^2 \begin{vmatrix} 0 & 0 & xy + yz + zx - x^2 - y^2 - z^2 \\ x-y & x-z & yz - x^2 \\ y-z & y-x & zx - y^2 \end{vmatrix} \quad 1$$

Expanding to get

$$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2 \quad 1$$

Hence  $\Delta$  is divisible by  $(x + y + z)$  and

$$\text{the quotient is } (x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2 \quad 1$$

**OR**

$$\text{Writing} \quad \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad 1$$

$$R_1 \leftrightarrow R_3 \quad \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$$

$$\begin{matrix} R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 - 4R_2 \end{matrix} \quad \begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$$

$$\begin{matrix} R_1 \rightarrow \frac{1}{3}R_1 \\ R_3 \rightarrow -R_3 \end{matrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$$

$$R_2 \rightarrow R_2 - 2R_1 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A \quad 2\frac{1}{2} \text{ marks for operation to get } A^{-1}$$

$$R_2 \rightarrow R_2 - R_3 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$$

$$\therefore A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \quad \frac{1}{2}$$

$$AX = B \Rightarrow X = A^{-1}B$$

1

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

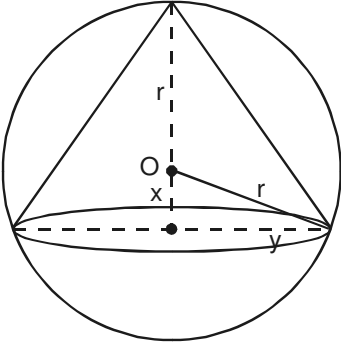
$$\therefore x = 1, y = 2, z = 1$$

1

25.

Correct Figure

1

Let radius of cone be  $y$  and the altitude be  $r + x$ 

$$\therefore x^2 + y^2 = r^2 \quad \dots(i)$$

 $\frac{1}{2}$ 

$$\text{Volume } V = \frac{1}{3} \pi y^2 (r + x)$$

$$= \frac{1}{3} \pi (r^2 - x^2) (r + x)$$

1

$$\frac{dV}{dx} = \frac{\pi}{3} [(r^2 - x^2)1 + (r + x)(-2x)] = \frac{\pi}{3} (r + x)(r - 3x)$$

1

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3}$$

 $\frac{1}{2}$ 

$$\therefore \text{Altitude} = r + \frac{r}{3} = \frac{4r}{3}$$

 $\frac{1}{2}$ 

$$\text{and } \frac{d^2V}{dx^2} = \frac{\pi}{3} [(r + x)(-3) + (r - 3x)] = \frac{\pi}{3} [-2r - 6x] < 0$$

1

$$\therefore \text{Max. Volume} = \frac{\pi}{3} \left( r^2 - \frac{r^2}{9} \right) \left( r + \frac{r}{3} \right) = \frac{8}{27} \left( \frac{4}{3} \pi r^3 \right)$$

 $\frac{1}{2}$ 

$$= \frac{8}{27} (\text{Vol. of sphere})$$

**OR**

$$f(x) = \sin 3x - \cos 3x, 0 < x < \pi$$

$$f'(x) = 3 \cos 3x + 3 \sin 3x$$

1

$$f'(x) = 0 \Rightarrow \tan 3x = -1$$

 $\frac{1}{2}$ 

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{4}, n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

 $\frac{1}{2}$ 

$$\text{Intervals are: } \left( 0, \frac{\pi}{4} \right), \left( \frac{\pi}{4}, \frac{7\pi}{12} \right), \left( \frac{7\pi}{12}, \frac{11\pi}{12} \right), \left( \frac{11\pi}{12}, \pi \right)$$

1

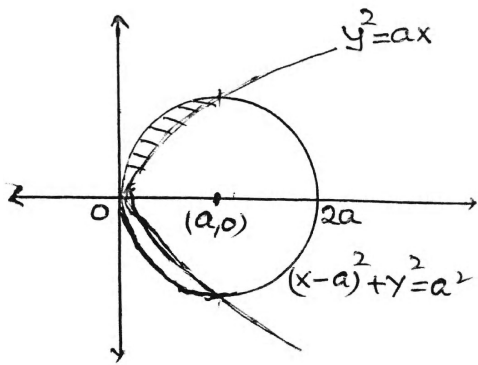
$$f(x) \text{ is strictly increasing in } \left( 0, \frac{\pi}{4} \right) \cup \left( \frac{7\pi}{12}, \frac{11\pi}{12} \right)$$

1

$$\text{and strictly decreasing in } \left( \frac{\pi}{4}, \frac{7\pi}{12} \right) \cup \left( \frac{11\pi}{12}, \pi \right)$$

1

26.



$$y^2 = ax, x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$$

$$\Rightarrow x = 0, x = a$$

1

Correct Figure

1

$$\text{Shaded area} = \left[ \int_0^a [\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}] dx \right]$$

1

$$A = \left[ \frac{x-a}{2} \sqrt{a^2 - (x-a)^2} + \frac{a^2}{2} \sin^{-1} \frac{x-a}{a} - \sqrt{a} \frac{2}{3} x^{3/2} \right]_0^a$$

2

$$= \left[ -\frac{2}{3} a^2 + \frac{a^2}{2} \frac{\pi}{2} \right] = \frac{\pi a^2}{4} - \frac{2a^2}{3} \text{ sq. units}$$

1